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## Mixed QCD-electroweak corrections to Higgs boson production in gluon fusion

Anastasiou, C ; Boughezal, R ; Petriello, F

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# Mixed QCD-Electroweak corrections to Higgs boson production in gluon fusion

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**Charalampos Anastasiou**

*Institute for Theoretical Physics, ETH Zurich,  
8093 Zurich, Switzerland  
E-mail: [babis@phys.ethz.ch](mailto:babis@phys.ethz.ch)*

**Radja Boughezal**

*Institute for Theoretical Physics, University of Zurich,  
Winterthurerstr. 190, 8057 Zurich, Switzerland  
E-mail: [radja@phys.uzh.ch](mailto:radja@phys.uzh.ch)*

**Frank Petriello**

*Department of Physics, University of Wisconsin,  
Madison, WI 53706 USA  
E-mail: [frankjp@physics.wisc.edu](mailto:frankjp@physics.wisc.edu)*

**ABSTRACT:** We compute the 3-loop  $\mathcal{O}(\alpha\alpha_s)$  correction to the Higgs boson production cross section arising from light quarks using an effective theory approach. Our calculation probes the factorization of QCD and electroweak perturbative corrections to this process. We combine our results with the best current estimates for contributions from top and bottom quarks to derive an updated theoretical prediction for the Higgs boson production cross section in gluon fusion. Our study leads to values for the cross section approximately 7 – 10% larger than those used in recent Tevatron analyses that imposed a 95% confidence level exclusion limit of a Standard Model Higgs boson with  $M_H = 170$  GeV.

**KEYWORDS:** NLO and NNLO computations, Higgs physics.

## 1. Introduction

The Higgs boson is the last undiscovered particle of the Standard Model. The search for the Higgs is a primary goal of the LHC program, and it is also a top priority at the Tevatron. A discovery of the Higgs boson is feasible with good confidence at the LHC for all mass values where the Standard Model remains consistent. The Tevatron experiments are becoming sensitive to Higgs signals in the  $H \rightarrow WW$  channel for masses near the threshold  $M_H \approx 2M_W$ . Recently, the Tevatron collaborations reported a 95% confidence level exclusion of a Standard Model Higgs boson with a mass near  $m_H = 170\text{ GeV}$  [1, 2].

Understanding the theoretical prediction is crucial to both the search for and exclusion of the Standard Model Higgs boson. Backgrounds to the Higgs signal are severe in many channels, particularly when a mass peak cannot be reconstructed such as in  $H \rightarrow WW \rightarrow l\nu l\nu$ , and knowledge of the signal shape and normalization is needed to optimize experimental searches. Measurements of Higgs boson couplings will also require the best possible theoretical predictions [3, 4]. The dominant production mode at both the Tevatron and the LHC, gluon fusion through top-quark loops, receives important QCD radiative corrections [5–7]. The inclusive result increases by a factor of 2 at the LHC and 3.5 at the Tevatron when perturbative QCD effects through next-to-next-to-leading order (NNLO) are taken into account [8–10]. The theoretical uncertainty from effects beyond NNLO is estimated to be about  $\pm 10\%$  by varying renormalization and factorization scales. A better perturbative convergence and a much smaller scale variation are found when typical experimental cuts which suppress associated jet radiation at high transverse momentum and enhance the  $H \rightarrow WW$  signal at Tevatron and the LHC are implemented [11–16].

The importance and success in taming the QCD corrections to Higgs production have shifted attention to electroweak corrections to the Higgs signal. The authors of Refs. [17, 18] pointed out important 2-loop light-quark effects; these are pictured in Fig. (1) of this manuscript and involve the Higgs coupling to  $W$ - or  $Z$ -bosons which then couple to gluons through a light-quark loop. These terms are not suppressed by light-quark Yukawa couplings, and receive a multiplicity enhancement from summing over the quarks. A careful study of the full 2-loop electroweak effects was performed in Ref. [19]. They increase the leading-order cross section by up to 5 – 6% for relevant Higgs masses. However, it is unclear whether these contributions receive the same large QCD enhancement as the top quark loops. If they do, then the full NNLO QCD result is shifted by +5 – 6% from these electroweak corrections. If not, this 5 – 6% increase from light quarks would be reduced to 1 – 2% of the NNLO result. As this effect on the central value of the production cross section and therefore on the exclusion limits and future measurements is non-negligible, it is important to quantify it. The exact computation of the mixed electroweak/QCD effects needed to do so requires 3-loop diagrams with many kinematic scales, and 2-loop diagrams with four external legs for the real-radiation terms. Such a computation is prohibitively difficult with current computational techniques.

In this paper we compute the QCD correction to the light-quark terms in the Higgs production cross section using an effective theory approach. This technique reduces the calculation to the evaluation of 3-loop vacuum bubbles. The effective theory is formally

valid only for  $M_H < M_W$ . However, there are reasons to believe that the  $K$ -factor computed with the effective theory has an extended range of validity. In the top-quark contribution to gluon fusion, the effective theory obtained after decoupling the top quark is formally valid only for Higgs boson masses  $M_H < 2m_t$ . Nevertheless, the  $K$ -factor obtained is an extremely good approximation to the exact one for Higgs boson masses up to  $m_H \approx 1$  TeV [5]. We find that the correction to the light-quark terms is not as large as those affecting the top-quark contribution. Nevertheless, the two corrections have the same sign, and the numerical effect of the difference is small, indicating that the 5 – 6% shift is indeed realized.

A second goal of this manuscript is to provide the most up-to-date QCD prediction for the Higgs boson production cross section in gluon fusion for use in setting Tevatron exclusion limits. The CDF and D0 collaborations currently use results from Ref. [20], which are several years old [21]. The following aspects of the analysis given must be updated to account for recent developments.

- Both CTEQ and MRST parton distribution functions (PDFs) have shifted significantly in the past several years due to an improved treatment of heavy-quark mass effects at low  $Q^2$  [22–24]. The new PDFs have an increased gluon distribution near the values of  $x$  relevant for Higgs boson production, and also require a larger value for  $\alpha_s(M_Z)$ . These changes increase the predicted production cross section.
- The analysis in Ref. [20] used the  $K$ -factor computed in the effective theory with the top quark integrated out for both the top- and bottom-quark contributions. The NLO QCD correction to the bottom-quark contribution is known to be smaller than the NLO top-quark  $K$ -factor [25]. This effect increases the predicted production cross section.
- The 2-loop light-quark contributions together with their QCD correction evaluated as discussed above were not known at the time of the previous computation.

We present results for the Higgs boson cross section accounting for these effects. We account for the effect of soft-gluon resummation at the Tevatron by presenting values for the scale choice  $\mu_F = \mu_R = M_H/2$ , which is known to very accurately reproduce the central value of the resummation result [20] for a wide range of Higgs boson masses, and provide an estimate of the remaining theoretical uncertainties arising from unknown higher-order terms and PDF errors. The updated numerical values for the cross section are approximately 7 – 10% higher than those used in Tevatron analyses, and motivate a reanalysis of the Tevatron exclusion limits.

Our paper is organized as follows. In Section 2 we describe our calculation of the 3-loop light-quark correction to the Higgs production cross section, detailing the formulation of the effective theory and technical aspects. In Section 3 we present numerics for both the light-quark electroweak shifts and the updated inclusive cross section. We conclude in Section 4.

## 2. Calculational Details

The cross section for Higgs boson production in hadronic collisions can be written as

$$\begin{aligned} \sigma(s, M_H^2) = & \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_{i/h_1}(x_1, \mu_F^2) f_{j/h_2}(x_2, \mu_F^2) \int_0^1 dz \delta\left(z - \frac{M_H^2}{x_1 x_2 s}\right) \\ & \times z \hat{\sigma}_{ij}(z; \alpha_s(\mu_R^2), \alpha_{EW}, M_H^2/\mu_R^2; M_H^2/\mu_F^2). \end{aligned} \quad (2.1)$$

Here,  $\sqrt{s}$  is the center-of-mass energy of the hadronic collision,  $\mu_R$  and  $\mu_F$  respectively denote the renormalization and factorization scales, and the  $f_{i/h}$  denote the parton densities. The quantity  $z\hat{\sigma}$  is the partonic cross section for the process  $ij \rightarrow H + X$  with  $i, j = g, q, \bar{q}$ . As indicated, it admits a joint perturbative expansion in the strong and electroweak couplings.

The leading term in the partonic cross section arises from a one-loop correction coupling the Higgs boson to gluons via a top-quark loop. Considering only QCD interactions for the moment and suppressing the scale dependences for simplicity, the partonic cross section can be written as

$$\hat{\sigma}_{ij} = \sigma^{(0)} G_{ij}(z; \alpha_s), \quad (2.2)$$

with

$$\sigma^{(0)} = \frac{G_F \alpha_s^2}{512 \sqrt{2} \pi} \left| \mathcal{G}_t \right|^2, \quad (2.3)$$

$$\mathcal{G}_q = -4 q_H \left[ 2 - (1 - 4 q_H) H \left( -r, -r; -\frac{1}{q_H} \right) \right], \quad (2.4)$$

$q_H = m_q^2/M_H^2$ , and

$$H(-r, -r; x) = \frac{1}{2} \ln^2 \left[ \frac{\sqrt{4+x} - \sqrt{x}}{\sqrt{4+x} + \sqrt{x}} \right]. \quad (2.5)$$

The contribution from bottom quarks in the Standard Model is also numerically relevant; we discuss its inclusion later in this paper. The coefficient functions can be expanded in the strong coupling constant  $\alpha_s$  as

$$G_{ij}(z; \alpha_s) = \sum_{n=0}^{\infty} \left( \frac{\alpha_s}{\pi} \right)^n G_{ij}^{(n)}(z), \quad (2.6)$$

with the leading term given by  $G_{ij}^{(0)}(z) = \delta_{ig} \delta_{jg} \delta(1-z)$ . The NLO coefficient functions have been computed in Ref. [5] retaining the exact dependence on the quark mass. The NNLO results in the large  $m_q$  limit, relevant when  $2m_q > M_H$ , were derived in Refs. [8–10]. The NLO result in this limit was obtained in Refs. [6, 7]. The QCD corrections have a large effect on the predicted cross section, increasing it roughly by a factor of 2 at the LHC and by a factor of 3.5 at the Tevatron.

Important electroweak corrections arise from two-loop diagrams containing an internal quark loop where the Higgs boson couples to  $W$ - and  $Z$ -bosons. An example diagram is shown in Fig. (1); we henceforth refer to these corrections as light-quark electroweak contributions, while the quark Yukawa coupling dependent terms discussed above are denoted

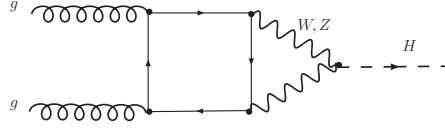
as heavy-quark QCD contributions. The light-quark diagrams are not suppressed by quark Yukawa couplings, and therefore have a multiplicity enhancement from summing over light quarks. The inclusion of these contributions modifies the term proportional to  $G_{ij}^{(0)}(z)$  in Eq. (2.2). The partonic cross section becomes

$$\hat{\sigma}_{ij} = \sigma_{\text{EW}}^{(0)} G_{ij}^{(0)}(z) + \sigma^{(0)} \sum_{n=1}^{\infty} \left( \frac{\alpha_s}{\pi} \right)^n G_{ij}^{(n)}(z) \quad (2.7)$$

with

$$\sigma_{\text{EW}}^{(0)} = \frac{G_F \alpha_s^2}{512 \sqrt{2} \pi} \left| \mathcal{G}_{lf}^{2l} + \mathcal{G}_t \right|^2. \quad (2.8)$$

$\mathcal{G}_{lf}^{2l}$  is the expression for the two-loop light-quark contributions; its analytic form in terms of generalized harmonic polylogarithms can be found in Ref. [17]. A careful numerical study of these electroweak corrections utilizing the complex-mass scheme to handle the threshold regions  $M_H \approx 2 M_{W,Z}$  was performed recently in Ref. [19]; this study also includes effects from internal top quarks coupling to the  $W$  and  $Z$ . The full corrections increase the leading-order cross section by  $+5 - 6\%$  for Higgs boson masses in the range  $120 - 160 \text{ GeV}$ .



**Figure 1:** Example two-loop light-quark diagram contributing to the Higgs boson production cross section via gluon fusion.

The cross section in Eq. (2.7) includes corrections to the leading-order result valid through  $\mathcal{O}(\alpha)$  in the electroweak couplings and to  $\mathcal{O}(\alpha_s^2)$  in the QCD coupling constant in the large top-mass limit upon inclusion of the known results for  $G_{ij}^{(1,2)}$ . Since the perturbative corrections to the leading-order result are large, it is important to quantify the effect of the QCD corrections on the light-quark electroweak contributions. This would require knowledge of the mixed  $\mathcal{O}(\alpha\alpha_s)$  corrections, which arise from 3-loop diagrams. In lieu of such a calculation, the authors of Ref. [19] studied two assumptions for the effect of QCD corrections on the 2-loop light-quark diagrams.

- *Partial factorization:* no QCD corrections to the light-quark electroweak diagrams are included, so that the cross section is given by the expression in Eq. (2.7). With this assumption, electroweak diagrams contribute only a  $+1 - 2\%$  increase to the Higgs boson production cross section.
- *Complete factorization:* the QCD corrections to the electroweak contributions are assumed to be identical to those affecting the heavy-quark diagrams, and the partonic cross section is therefore taken to be

$$\hat{\sigma}_{ij}^{CF} = \sigma_{\text{EW}}^{(0)} G_{ij}(z; \alpha_s) \quad (2.9)$$

with the full QCD coefficient function multiplying both the heavy- and light-quark contributions. In this case the light-quark diagrams increase the full NNLO QCD production cross section by  $+5 - 6\%$ .

The resulting shift in the central value for the Higgs boson production cross section can have a non-negligible effect on exclusion limits at the Tevatron, and on future exclusion limits or measurements of Higgs boson properties at the LHC.

We discuss later in this manuscript the inclusion of bottom-quark contributions to the Higgs production cross section. We define for future reference the following point-like cross sections:

$$\begin{aligned}\sigma_b^{(0)} &= \frac{G_F \alpha_s^2}{512\sqrt{2}\pi} |\mathcal{G}_b|^2, \\ \sigma_{t,b}^{(0)} &= \frac{G_F \alpha_s^2}{512\sqrt{2}\pi} [2 \operatorname{Re} (\mathcal{G}_t \mathcal{G}_b^*)], \\ \sigma_{t,lf}^{(0)} &= \frac{G_F \alpha_s^2}{512\sqrt{2}\pi} [2 \operatorname{Re} (\mathcal{G}_t \mathcal{G}_{lf}^*)].\end{aligned}\tag{2.10}$$

$\sigma_b^{(0)}$  denotes the squared bottom-quark loop,  $\sigma_{t,b}^{(0)}$  the interference between the top and bottom loops, and  $\sigma_{t,lf}^{(0)}$  the interference between the top-quark contribution and the light-quark terms.

## 2.1 The Effective Lagrangian formulation

A rigorous test of factorization of QCD and electroweak corrections to Higgs boson production in the Standard Model for all values of  $M_H$  would require a full 3-loop calculation containing several mass scales. Such a computation is seemingly beyond current technical capabilities. However, in the limit  $M_H < M_{W,Z}$ , an approximate result can be obtained by expanding around the point  $M_H/M_{W,Z} = 0$ . This is the same approach used to derive the heavy-quark result when  $M_H/(2m_t) < 1$ . Although experimentally  $M_H > M_{W,Z}$ , it is known that the approximate NLO correction to the heavy-quark result matches the exact NLO correction extremely well up to  $M_H \approx 1 \text{ TeV}$  due to the structure of the QCD corrections. This provides some reason to believe the same is true for the light-quark contributions.

The  $M_H = 0$  expansion is most clearly formulated using an effective Lagrangian, and we review this approach here. The leading term in the expansion for the top-quark contribution in Eqs. (2.3) and (2.4) can be obtained via Feynman rules derived from

$$\mathcal{L}_{eff} = -\alpha_s \frac{C_1}{4v} G_{\mu\nu}^a G^{a\mu\nu}.\tag{2.11}$$

The Wilson coefficient arising from integrating out the heavy quark is

$$\begin{aligned}C_1 &= -\frac{1}{3\pi} \{1 + a_s C_{1q} + a_s^2 C_{2q} + \mathcal{O}(a_s^3)\}, \\ C_{1q} &= \frac{11}{4}, \quad C_{2q} = \frac{2777}{288} + \frac{19}{16} L_t + N_F \left( -\frac{67}{96} + \frac{1}{3} L_t \right),\end{aligned}\tag{2.12}$$

where  $a_s = \alpha_s/\pi$ ,  $N_F = 5$  is the number of active quark flavors and  $L_t = \ln(\mu_R^2/m_t^2)$ . We now include the leading term in the  $M_H/M_{W,Z}$  expansion in the Wilson coefficient. It can be obtained by expanding  $\mathcal{G}_{lf}^{2l}$  given in Ref. [17]; denoting the contribution by  $\lambda_{EW}$ , we find

$$\begin{aligned} C_1 &= -\frac{1}{3\pi} \left\{ 1 + \lambda_{EW} + a_s C_{1q} + a_s^2 C_{2q} + \mathcal{O}(a_s^3) \right\}, \\ \lambda_{EW} &= \frac{3\alpha}{16\pi s_W^2} \left\{ \frac{2}{c_W^2} \left[ \frac{5}{4} - \frac{7}{3} s_W^2 + \frac{22}{9} s_W^4 \right] + 4 \right\}, \end{aligned} \quad (2.13)$$

where  $s_W, c_W$  are respectively the sine and cosine of the weak-mixing angle.

The QCD corrections modify the Wilson coefficient to include terms of  $\mathcal{O}(\lambda_{EW} a_s)$  and  $\mathcal{O}(\lambda_{EW} a_s^2)$ . The extent to which factorization of electroweak and QCD corrections holds becomes a question regarding to what extent the Wilson coefficient can be written as a product of separate QCD and electroweak factors. We denote the exact coefficients of these terms as  $C_{1w}$  and  $C_{2w}$  respectively, and introduce below in Eq. (2.14) the exact Wilson coefficient and the factorized hypothesis:

$$\begin{aligned} C_1 &= -\frac{1}{3\pi} \left\{ 1 + \lambda_{EW} [1 + a_s C_{1w} + a_s^2 C_{2w}] + a_s C_{1q} + a_s^2 C_{2q} \right\}, \\ C_1^{fac} &= -\frac{1}{3\pi} (1 + \lambda_{EW}) \left\{ 1 + a_s C_{1q} + a_s^2 C_{2q} \right\}. \end{aligned} \quad (2.14)$$

Factorization holds if  $C_{1w} = C_{1q}$  and  $C_{2w} = C_{2q}$ . We will derive here the  $C_{1w}$  coefficient by expanding the 3-loop QCD corrections to the light-quark electroweak diagrams to test this. We do not compute  $C_{2w}$ , but will study later the numerical effect of various choices for this term.

## 2.2 Calculational approach

We begin by generating all 3-loop diagrams for  $g(p_1) + g(p_2) \rightarrow H(p_H)$  containing two internal  $W$ - or  $Z$ -boson propagators coupling to a light-quark loop. For the  $Z$ -boson and for a single quark flavor, there are 51 such non-vanishing diagrams. Examples are shown in Fig. (2). The only difference for the  $W$ -boson is the change of quark flavor at the vertex. All such details are accounted for in  $\lambda_{EW}$  and do not affect the computation of  $C_{1w}$ . We perform a Taylor expansion of the integrand of each diagram in the external momenta  $p_{1,2}$ . This is most conveniently performed by applying the following differential operator to each diagram [27]:

$$\mathcal{D}\mathcal{F} = \sum_{n=0}^{\infty} (p_1 \cdot p_2)^n [D_n \mathcal{F}]_{p_1=p_2=0} \quad (2.15)$$

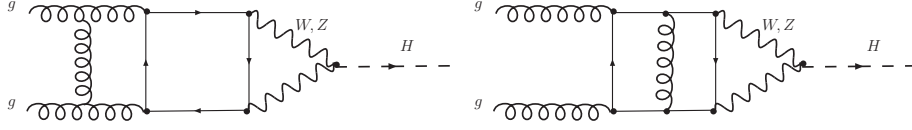
where  $\mathcal{F}$  denotes a diagram. The first few terms in the sum are

$$D_0 = 1, \quad D_1 = \frac{1}{d} \square_{12}, \quad D_2 = -\frac{1}{2(d-1)d(d+2)} \left\{ \square_{11} \square_{22} - d \square_{12}^2 \right\}, \quad (2.16)$$

where  $\square_{ij} = \frac{\partial}{\partial p_{i\mu}} \frac{\partial}{\partial p_j^\mu}$ . The amplitude arising from summing over Feynman diagrams can be written as

$$\sum \mathcal{F} = \mathcal{A} \left\{ g_{\mu\nu} - \frac{p_{2\mu} p_{1\nu}}{p_1 \cdot p_2} \right\} \delta^{ab} \epsilon_a^\mu(p_1) \epsilon_b^\nu(p_2) \equiv \mathcal{M}_{\mu\nu}^{ab} \epsilon_a^\mu(p_1) \epsilon_b^\nu(p_2). \quad (2.17)$$





**Figure 2:** Example three-loop light-quark diagrams contributing to the  $C_{1w}$  term in the Wilson coefficient. We note that the left diagram has a cut at  $p_H^2 = 0$ ; the right diagram develops an imaginary part at  $p_H^2 = M_{W,Z}^2$ .

The coefficient  $\mathcal{A}$  can be obtained by acting with the appropriate projection operator:

$$\mathcal{A} = \frac{1}{8(d-2)} \left\{ g^{\mu\nu} - \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1 \cdot p_2} \right\} \delta_{ab} \mathcal{M}_{\mu\nu}^{ab}. \quad (2.18)$$

The leading term in the expansion of  $\mathcal{A}$  must be finite, and gives  $C_{1w}$  upon comparison with Eqs. (2.11) and (2.14). Sub-leading terms in the Taylor expansion need not be finite, as 3-loop diagrams with a cut at  $p_H^2 = 0$  that generate logarithms in  $m_H$  exist; an example is the left-most diagram in Fig. (2). We note that the 2-loop light-quark contribution and the 3-loop diagrams contributing to  $C_{1w}$  have a cut first at  $p_H^2 = M_{W,Z}^2$  because of helicity flow along the internal quark line [26], indicating that the radius of convergence of the expansion is  $M_H < M_{W,Z}$ .

All integrals appearing in the calculation of  $\mathcal{A}$  can be mapped to the following topology:

$$\begin{aligned} \mathcal{I}(\nu_1, \nu_2, \nu_3, \nu_4, \nu_5, \nu_6) &= \int d^d k_1 d^d k_2 d^d k_3 \frac{1}{[k_1^2]^{\nu_1} [k_2^2]^{\nu_2} [k_3^2 - M_{W,Z}^2]^{\nu_3}} \\ &\times \frac{1}{[(k_1 - k_2)^2]^{\nu_4} [(k_2 - k_3)^2]^{\nu_5} [(k_3 - k_1)^2]^{\nu_6}}. \end{aligned} \quad (2.19)$$

These can be reduced to a small set of master integrals using what is by now standard technology based on the solution of integration-by-parts identities [28–30]. We find that only the integrals  $\mathcal{I}(1, 0, 1, 1, 1, 0)$  and  $\mathcal{I}(1, 1, 1, 0, 1, 1)$  are needed to describe this process. It is straightforward to express both as a simple product of Gamma functions.

After a computation following the approach outlined above, we obtain the primary analytic result of this paper:

$$C_{1w} = \frac{7}{6}. \quad (2.20)$$

Two points should be noted regarding the comparison of this with the factorization hypothesis  $C_{1w}^{fac} = C_{1q} = 11/4$ . First, there is a fairly large violation of the factorization result:  $(C_{1q} - C_{1w})/C_{1w} \approx 1.4$ . However, both expressions have the same sign, and a large difference from the +5–6% shift found before does not occur. We study in the next section the numerical effect of  $C_{1w}$  and the unknown  $C_{2w}$ .

### 3. Numerical Results

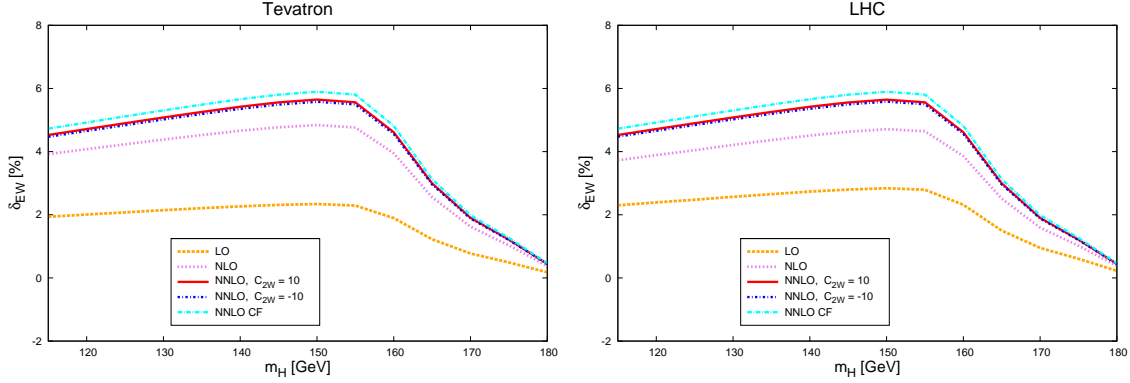
We present here numerical results for the Higgs production cross section and the shift arising from light-quark electroweak diagrams including the effect of  $C_{1w}$  and  $C_{2w}$ . Our

purpose is two-fold: to study the numerical effect of the correction computed in the previous section, and to provide an updated prediction for the inclusive cross section for use in experimental studies. We include all currently computed perturbative effects on the cross section. These include the NNLO  $K$ -factor computed in the large- $m_t$  limit and normalized to the exact  $m_t$ -dependent LO result, the full light-quark electroweak correction and the  $\mathcal{O}(\alpha_s)$  correction to this encoded in  $C_{1w}$ , and the bottom-quark contribution. We define the following cross sections for use in our discussion, together with the PDFs used to obtain hadronic results from each:

$$\begin{aligned}
\sigma_{QCD}^{NNLO} &= \sigma^{(0)} G_{ij}(z; \alpha_s) + \sigma_b^{(0)} G_{ij}^{(0)}(z) K_{bb} + \sigma_{t,b}^{(0)} G_{ij}^{(0)}(z) K_{tb} \quad (\text{MRST2006 NNLO}), \\
\sigma_{EW}^{LO} &= \sigma_{t,l f}^{(0)} G_{ij}^{(0)}(z) \quad (\text{MRST2001 LO}), \\
\sigma_{EW}^{NLO} &= \sigma_{t,l f}^{(0)} \left\{ G_{ij}^{(0)}(z) [1 + a_s(C_{1w} - C_{1q})] + a_s G_{ij}^{(1)}(z) \right\} \quad (\text{MRST2004 NLO}), \\
\sigma_{EW}^{NNLO} &= \sigma_{t,l f}^{(0)} \left\{ G_{ij}^{(0)}(z) [1 + a_s(C_{1w} - C_{1q}) + a_s^2(C_{2w} - C_{2q} + C_{1q}(C_{1q} - C_{1w}))] \right. \\
&\quad \left. + a_s G_{ij}^{(1)}(z) [1 + a_s(C_{1w} - C_{1q})] + a_s^2 G_{ij}^{(2)}(z) \right\} \quad (\text{MRST2006 NNLO}), \\
\sigma_{EW}^{NNLO CF} &= \sigma_{t,l f}^{(0)} G_{ij}(z; \alpha_s) \quad (\text{MRST2006 NNLO}), \\
\sigma^{best} &= \sigma_{QCD}^{NNLO} + \sigma_{EW}^{NNLO} \quad (\text{MRST2006 NNLO}).
\end{aligned} \tag{3.1}$$

We briefly describe here the content of these several terms.  $\sigma_{QCD}^{NNLO}$  includes contributions from both top- and bottom-quark loops, with  $\sigma_b^{(0)}$  and  $\sigma_{t,b}^{(0)}$  defined in Eq. (2.10). The QCD corrections to the top-quark in the large- $m_t$  limit are encoded in  $G_{ij}(z; \alpha_s)$ . The NLO  $K$ -factors for the squared bottom-quark term  $\sigma_b^{(0)}$  and the interference term  $\sigma_{t,b}^{(0)}$  as derived from Ref. [25] are included in  $K_{bb}$  and  $K_{tb}$  respectively. We note that both the study in Ref. [20] and the Tevatron analysis put the bottom-quark terms in  $\sigma^{(0)}$ , and therefore use the same  $K$ -factor for both top- and bottom-quark loops. This results in a significant underestimate of the cross section, since the effect of these terms is negative; while the NNLO  $K$ -factor for the top-quark term is roughly 3.5 and the NLO  $K$ -factor is over 2,  $K_{bb}$  and  $K_{tb}$  only vary between 1.4 and 1.7 for Higgs boson masses between 120 – 180 GeV.

The remaining terms in Eq. (3.1) denote the contributions from light-quark diagrams in various approximations.  $\sigma_{t,l f}^{(0)}$  denotes the interference between the dominant top-quark term and the light quarks defined in Eq. (2.10); in our numerics we use the exact results of Ref. [19] which are valid for arbitrary Higgs boson masses. The squared light-quark term is numerically irrelevant.  $\sigma_{EW}^{LO}$  includes only the 2-loop diagrams computed in Refs. [17, 19] and is equivalent to the partial factorization hypothesis defined in Sec. 2.  $\sigma_{EW}^{NLO}$  includes the  $\mathcal{O}(\alpha_s)$  correction to these diagrams computed in the effective theory and encoded in  $C_{1w}$ .  $\sigma_{EW}^{NNLO}$  includes the full  $\mathcal{O}(\alpha_s^2)$  correction to the light-quark diagrams including the unknown coefficient  $C_{2w}$ . We study numerically below various choices for  $C_{2w}$ .  $\sigma_{EW}^{NNLO CF}$  is the complete factorization hypothesis defined in Sec. 2. Finally,  $\sigma^{best}$  is the current best prediction for the Higgs boson cross section including all effects of top, bottom, and light quarks with the best estimates of their associated QCD corrections. To approximately implement the effects of soft-resummation, we make the scale choice  $\mu_R = \mu_F = M_H/2$ . This choice is known to reproduce the central value of the resummation results to 1%



**Figure 3:** Relative shifts to the Higgs boson production cross section at the Tevatron (left panel) and LHC (right panel) arising from light-quark diagrams. All curves are normalized to the full NNLO top-quark cross section and are produced for renormalization and factorization scales  $\mu_R = \mu_F = m_H/2$ . The various lines are described in detail in the accompanying text.

accuracy [20]. We evaluate the electroweak corrections using  $G_F$ ,  $M_W$  and  $M_Z$  as input parameters. We use the pole mass  $m_t = 170.9$  GeV for the top quark and the  $\overline{MS}$  mass  $\bar{m}_b(\mu_R)$  for the  $b$ -quark with the input value  $\bar{m}_b(10 \text{ GeV}) = 3.609$  GeV [31]. The choice of pole or  $\overline{MS}$  mass for the top quark has a negligible effect on the numerical results.

We begin by studying the percentage shifts arising from electroweak effects on the Higgs boson production cross section at both the Tevatron and the LHC in Fig. (3). The results shown in these plots are  $\delta_{EW}^x = 100 \times \sigma_{EW}^x / \sigma_{QCD}^{NNLO}$ , with the cross sections defined in Eq. (3.1). The close agreement between  $\delta_{EW}^{NNLO}$  and  $\delta_{EW}^{NNLOCF}$  occurs because the differences  $C_{1w} - C_{1q}$  and  $C_{2w} - C_{2q}$  in Eq. (3.1) are small compared to the effects of  $G_{ij}^{(1,2)}$  in  $\sigma_{EW}^{NNLO}$ . Furthermore, the unknown  $C_{2w}$  coefficient does not significantly alter the size of the electroweak shift. Our calculation confirms that the Higgs boson production cross section receives almost the entire numerical correction indicated by the complete factorization hypothesis.

We now combine all effects into a best prediction for the Higgs boson production cross section,  $\sigma^{best}$  defined in Eq. (3.1). We set  $C_{2w} = 0$  to produce these numbers. As discussed previously, several updates must be performed to the analysis in Ref. [20] and therefore the Tevatron exclusion limits.

- The  $K$ -factors for  $\sigma_b^{(0)}$  and  $\sigma_{t,b}^{(0)}$  are now known to be significantly smaller than those for top-quark term  $\sigma^{(0)}$  [25]. For example, in the pole-mass scheme the NLO  $K$ -factor for the top-bottom interference is approximately 1.5 for  $M_H = 150$  GeV, as compared to over 2 at NLO and 3.5 at NNLO for the top-quark term; the  $K$ -factor for the bottom-quark piece is roughly 1.7 for this Higgs mass. The study in Ref. [20] utilized the top-quark  $K$ -factor for all three terms. As mentioned above we use the  $\overline{MS}$   $b$ -quark mass instead of the pole mass of Ref. [20]. Our final cross section numbers with the  $\overline{MS}$   $b$ -mass are about 1.5% larger than the numbers computed in the pole scheme. Although the results in both schemes are very similar after

original	MRST 2006 PDFs	$K_{tb}, K_{bb}$	EW effects
0.3542	0.3650	0.3868	0.3943

**Table 1:** Shifts in the Higgs boson production cross section resulting from the changes detailed in the text. All numbers are in picobarns.

the inclusion of NLO corrections, previous studies have shown some preference for a running mass [5, 32] so we present numbers for this scheme.

- Updated PDF extractions by CTEQ and MRST with an improved treatment of heavy-quark effects at low  $Q^2$  have a significant effect on the cross section.
- The electroweak terms derived above must be added.

These corrections all have an important effect on the predicted cross section. For illustration, we show below the sequential effect of making these changes on the cross section for  $M_H = 170$  GeV. We begin by reproducing the  $\mu_F = \mu_R = M_H/2$  numbers given in Ref. [20], which also matches the reference value for the resummed result, by implementing MRST 2002 NNLO PDFs, using the masses  $m_t = 176$  GeV and  $m_b = 4.75$  GeV, multiplying all top- and bottom-quark terms by the  $K$ -factor appropriate for the top quark, and removing the 2-loop light-quark terms. We then perform the following changes: (1) we switch to MRST 2006 NNLO PDFs; (2) we switch to the current extracted top-quark mass  $m_t = 170.9$  GeV, and to  $\bar{m}_b$ , and use the NLO  $K_{tb}$  and  $K_{bb}$  respectively to model the QCD corrections to the top-bottom interference and the bottom-quark squared contribution; (3) we implement the electroweak corrections described above. The results are shown in Table 1. While each effect is individually small, they act constructively to increase the cross section by over 10%. We apply all of these corrections to provide up-to-date values for the inclusive Higgs boson production cross section in Table 2 for the scale choice  $\mu = M_H/2$  that accurately reproduces the effect of soft-gluon resummation. These numerical values are 7 – 10% higher than values previously used by the Tevatron collaborations to establish exclusion limits on the Standard Model Higgs boson, and motivate a reconsideration of their results.

We also estimate the current theoretical uncertainties arising from uncalculated higher-order terms and PDF errors. To estimate the errors from higher-order effects we vary the scale  $\mu$  in the range  $[M_H/4, M_H]$ , which is a factor of two around the central value  $\mu_R = \mu_F = \mu = M_H/2$ . For the PDF errors we use the error eigenvectors provided with the MRST 2006 fit. The scale errors are constant with Higgs mass to very good approximation, and are  $[-11\%, +7\%]$ . The PDF uncertainties have a slight dependence on the Higgs boson mass; they are  $\pm 5\%$  in the mass range  $M_H = [150, 180]$  GeV, approximately  $\pm 4\%$  below this range, and  $[-5\%, +6\%]$  above  $M_H = 180$  GeV.

## 4. Conclusions

In this paper, we considered mixed QCD-electroweak corrections to the Higgs boson pro-

$m_H[\text{GeV}]$	$\sigma^{best}[\text{pb}]$	$m_H[\text{GeV}]$	$\sigma^{best}[\text{pb}]$
110	1.538	160	0.4928
115	1.354	165	0.4393
120	1.197	170	0.3943
125	1.062	175	0.3562
130	0.9459	180	0.3222
135	0.8447	185	0.2916
140	0.7566	190	0.2646
145	0.6794	195	0.2413
150	0.6113	200	0.2210
155	0.5506	—	—

**Table 2:** Higgs production cross section for Higgs mass values relevant for Tevatron, with  $\mu = \mu_R = \mu_F = M_H/2$ . The total cross section  $\sigma^{best}$  is defined in Eq. (3.1). The theoretical errors arising from higher-order terms and PDF uncertainties are detailed in the text.

duction cross section in the gluon-fusion channel. Working in an effective field theory valid for  $m_H < M_W$ , we provided the leading term of the three-loop  $\mathcal{O}(\alpha\alpha_s)$  contributions due to diagrams containing light quarks. This result allows us to check the factorization of electroweak and QCD corrections proposed in Ref. [18, 19]. We showed that, despite a fairly large violation of the factorization hypothesis, a significant numerical difference from the prediction of this hypothesis is not observed due to the structure of the QCD corrections. We combined the 2-loop light-quark diagrams based on the complex-mass scheme for the  $W$ - and  $Z$ -bosons [19] with our new 3-loop  $\mathcal{O}(\alpha\alpha_s)$  correction and with contributions from top and bottom quarks to provide an updated theoretical prediction for the production cross section. We found values that are 7 – 10% larger than those currently used by the Tevatron collaborations in the analysis that led to the 95% confidence level exclusion on a Standard Model Higgs boson with  $m_H = 170$  GeV. Our results motivate a reconsideration of the Tevatron exclusion limits.

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